PG – 253

II Semester M.Sc. Degree Examination, June/July 2014
(RNS)
(2011-12 & Onwards)
MATHEMATICS
M-202 : Complex Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer **any five full** question choosing atleast **two** from **each** Part.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) Define Harmonic function and evaluate $\int_c \frac{e^z}{z(z-1)(z-2)} dz$ where $c : |z| = 2$. **3**

b) Define conformal mapping and show that let z_1, z_2, z_3, z_4 be distinct points in $\overline{\mathbb{C}}$. Then (z_1, z_2, z_3, z_4) is real if and only if all four points lie on a circle. **7**

c) State and prove Cauchy's integral formula and use it to evaluate

$$\int_{|z|=3} \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)} dz.$$ **6**

2. a) State and prove Cauchy's theorem for a triangle. **6**

b) Let f (z) be analytic in a region G with zeros $a_1, a_2, ..., a_m$ repeated according to multiplicity. If r is a simple closed curve in G which does not pass through any a_k, then, prove that $\frac{1}{2\pi i} \int_r \frac{f'(z)}{f(z)} dz = \sum_{k=1}^{m} n(r : a_k)$ **7**

c) Evaluate $\int_r f(z)dz$ if $f(z) = \text{Re}(z)$ and r is the polygonal arc connecting 0 to 1 and 1 to 1 + i. **3**

P.T.O.

3. a) Find the radius of convergence of

i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n} (z-2i)^n$

ii) Prove that $1+\frac{a.\,b}{1.\,c}z+\frac{a\,(a+1)\,b\,(b+1)}{1.2.\,c\,(c+1)}\,z^2+\ldots$ has unit radius of convergence. **6**

b) Define radius of convergence of power series. Let $f(z)=\sum_{n=0}^{\infty} a_n(z-a)^n$ in $\{|z-a|<R\}$ where R is radius of convergence of the power series. Then prove that the Taylor's expansion of f(z) in the neighbourhood of a point 'a' is exactly the given power series. **6**

c) Show that $e^{\frac{1}{2}c(z-\frac{1}{z})}=\sum_{n=-\infty}^{\infty} a_n z^n$ where $a_n=\frac{1}{2\pi}\int_0^{2\pi}\cos(n\theta-c\sin\theta)\,d\theta$. **4**

4. a) State and prove Laurent's theorem. **6**

b) Let f(z) be analytic function having an isolated singularity at z = a. If |f(z)| is bounded in a neighbourhood $\{0<|z-a|<r\}$ then prove that f(z) has a removable singularity at z = a. **6**

c) Define the terms :

i) Pole

ii) Removable singularity

iii) Essential singularity

iv) Isolated singularity and give one example each. **4**

PART – B

5. Evaluate the following :

a) $\int_{0}^{2\pi} \frac{d\theta}{a + b\sin\theta}$

b) $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$

c) $\int_{-\infty}^{\infty} \frac{\cos(ax)}{(x^2 + b^2)^2} dx,\ a > 0$

d) $\int_{-\infty}^{\infty} e^{-x^2} \cos(2mx)\, dx,\ m > 0$ **(4+4+4+4=16)**

6. a) State and prove the argument principle theorem. **8**

   b) Show that $p(z) = e^z - 4z^2 - 1$ has exactly two roots in $| z | < 1$. **8**

7. a) State and prove the Schwartz Lemma. **8**

   b) State and prove the Hadmard's three circle theorem. **8**

8. a) State and prove Riemann mapping theorem. **8**

   b) State and prove Poisson's integral theorem. **8**